GAMMA-RAY ABSORPTION MECHANISMS

4–1 INTRODUCTION

In this chapter, we wish to discuss the various processes that are of astrophysical importance in depleting γ-rays in the galaxy and the universe. By "absorption" we will mean not only those processes in which the γ-ray completely disappears, such as pair-production, but also those processes in which the γ-ray is scattered out of the energy range of interest, as can occur in the case of Compton scattering.

We will consider two basic categories of absorption processes: Absorption of γ-rays in matter and absorption of γ-rays through interactions with radiation. The latter process is of importance because of the recently discovered 2.7 K blackbody radiation field which is now believed to fill the universe and to be a remnant of the initial cosmological "big-bang."

4–2 ABSORPTION OF GAMMA RAYS THROUGH INTERACTIONS WITH RADIATION

Let us first consider the effects of the universal radiation field on the intensity of cosmic γ-rays. The attenuation process of importance here is the pair-production process

$$\gamma + \gamma_{bb} \rightarrow e^+ + e^- \tag{4–1}$$

where γ represents the cosmic γ-ray and γ_{bb} the low-energy blackbody photon with which it interacts. This process can only take place if the total energy of the γ-rays in the cms of the interaction is greater than or equal to $2m_ec^2$. The cross section for reaction (4–1) can be calculated using quantum electrodynamics and a derivation may be found in Jauch and Rohrlich (1955). The importance of reaction (4–1) in astrophysics was first pointed out by Nikishov (1961) in considering interactions of γ-rays with ambient starlight photons and with the discovery of the universal thermal radiation. Gould and Schréder (1966, 1967a, b) and Jelly (1966) were quick to point out the opacity of the universe to γ-rays

of energy above 10^{14} eV. Stecker (1969c) and Fazio and Stecker (1970) generalized these calculations by including cosmological effects at high redshifts. (See ch. 13.)

In discussing reaction (4–1), we will generally follow the discussion of Gould and Schréder (1967a, b) with one important difference: At the time Gould and Schréder published their papers, it was generally thought that the universal blackbody radiation field was at a temperature of 3.0 K. More recent measurements (Stokes, Partridge, and Wilkinson, 1967) have yielded a temperature of 2.7 K for the blackbody radiation with a corresponding photon energy density of 0.25 eV/cm^3 instead of the value of 0.38 eV/cm^3 used by Gould and Schréder. Therefore, we have corrected the Gould and Schréder results to correspond to a 2.7 K radiation field.

In order to determine the threshold energy for reaction (4–1), we make use of the same procedure used in the derivation of equation (1–59) and start with the four-momentum invariance given by equation (1–52). Again, if we denote cms quantities by a prime and note that

$$E'_{e^+} = E'_{e^-} \equiv E'_e \tag{4–2}$$

equation (1–52) reduces to

$$\begin{aligned}(2E'_e)^2 &= (E_{\mathrm{bb}} + E_\gamma)^2 - |(\mathbf{p}_{\mathrm{bb}} + \mathbf{p}_\gamma)|^2 \\ &= (E^2_{\mathrm{bb}} + 2E_{\mathrm{bb}}E_\gamma + E^2_\gamma) - (E^2_{\mathrm{bb}} + E^2_\gamma - 2E_{\mathrm{bb}}E_\gamma \cos\theta) \\ &= 2E_{\mathrm{bb}}E_\gamma(1 - \cos\theta)\end{aligned} \tag{4–3}$$

To determine the threshold energy for reaction (4–1), we only need to specify that at this energy both the electron and the positron are produced at rest in the cms of the interaction. The minimum energy required is the one which corresponds to a head-on collision ($\cos\theta = -1$). Equation (4–3) then reduces to the relation specifying the threshold energy as

$$E_{\gamma,\,\mathrm{th}} = \frac{m_e^2}{E_{\mathrm{bb}}} \tag{4–4}$$

where, as in chapter 1, we have used the convention $c = 1$. It is interesting to note here the similarity between equation (4–4) and equation (1–97) for neutral pion decay. This is no accident and can be seen by picturing the pion decay run backward; however, instead of a pion, we picture the reverse decay of a particle made up of an electron and a positron stuck together, with a total rest mass of $2m_e$.

If we consider a typical blackbody photon to have an energy of approximately 10^{-9} MeV, then from equation (4–4) we find a threshold energy of approximately 2.5×10^{8} MeV for reaction (4–1). However, this threshold is somewhat blurred because the blackbody photons are not all at the same energy, but have a Bose-Einstein distribution in energy given by the well-known formula

$$n(E_{\mathrm{bb}}) = \frac{1}{\pi^2 \hbar^3 c^3} \frac{E_{\mathrm{bb}}^2}{1 - \exp\ (E_{\mathrm{bb}}/kT)} \tag{4–5}$$

We must also allow for the various possible values of cos θ given by the isotropic distribution (1–68) as

$$f(\theta) = \tfrac{1}{2} \sin\ \theta \tag{4–6}$$

The cross section for (4–1) is given by Jauch and Rohrlich (1955)

$$\sigma(E_{\mathrm{bb}}, E_\gamma) = \tfrac{1}{2}\sigma_0(1-\beta_e'^2)\ \left[(3-\beta_e'^4)\ \ln \frac{1+\beta_e'}{1-\beta_e'} - 2\beta_e'(2-\beta_e'^2)\right] \tag{4–7}$$

where the cms velocity of the electron (or positron) is given by

$$\beta_e' = \left(1 - \frac{m_e^2}{E_{\mathrm{bb}} E_\gamma}\right)^{1/2} \tag{4–8}$$

The absorption coefficient for γ-radiation against pair-production by interactions with the blackbody radiation is given by

$$\kappa_{\gamma\gamma}(E_\gamma) = \iint dE_{\mathrm{bb}}\ d\theta\ \frac{\sin\ \theta}{2}\ n(E_{\mathrm{bb}})\sigma(E_{\mathrm{bb}}, E_\gamma)(1 - \cos\ \theta) \tag{4–9}$$

Gould and Schréder (1967) have reduced equation (4–9) to the form

$$\kappa_{\gamma\gamma}(E_\gamma) = \alpha^3(\pi r_0)^{-1}\left(\frac{kT}{mc^2}\right)^3 f(\nu) \tag{4–10}$$

where

$$\nu \equiv \frac{(mc^2)^2}{E_\gamma kT} \tag{4–11}$$

They find that the function $f(\nu)$ has a maximum value ≈ 1 at $\nu \approx 1$ and that $f(\nu)$ has the asymptotic forms given by

$$f(\nu) \rightarrow \tfrac{1}{3}\pi^3 \nu\ \ln\ (0.117\nu) \qquad \text{for } \nu \ll 1$$

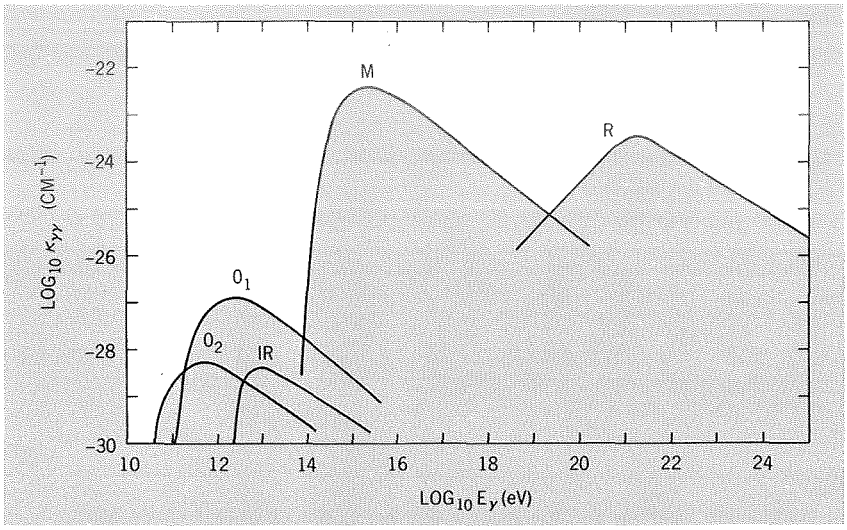

FIGURE 4–1.—Absorption probability per unit distance by $\gamma+\gamma' \rightarrow e^+ + e^-$ as a function of photon energy for γ-rays traversing the cosmic-photon gas. The contributions from the optical (O_1 and O_2), infrared (IR), microwave (M), and radio (R) cosmic-photon gas are shown. Absorption at lower energies by X-ray photons is negligible. O_1 represents the contribution from the light of population II stars; O_2 that from population I stars (from Gould and Schréder, 1967a, b); M is corrected to 2.7 K.

and

$$f(\nu) \rightarrow (\tfrac{1}{4}\pi\nu)^{1/2}e^{-\nu}\left(1+\frac{75}{8\nu}+\cdots\right) \qquad \text{for } \nu \gg 1 \qquad (4\text{–}12)$$

They have also calculated $\kappa_{\gamma\gamma}$ for γ-ray interactions with various photon fields in intergalactic space. The results of their numerical calculation are shown in figure 4–1.

4–3 ABSORPTION OF GAMMA RAYS THROUGH INTERACTIONS WITH MATTER

There are two types of interactions of importance to be considered here. The first is the Compton scattering interaction

$$\gamma + e^- \rightarrow \gamma + e^- \qquad (4\text{–}13)$$

whose cross section is given by the Klein-Nishima formula

$$\sigma_c(E_\gamma) = 2\sigma_0\left\{\frac{1+\epsilon}{\epsilon^2}\left[\frac{2(1+\epsilon)}{1+2\epsilon}-\frac{1}{\epsilon}\ln(1+2\epsilon)\right]+\frac{1}{2\epsilon}\ln(1+2\epsilon) - \frac{1+3\epsilon}{(1+2\epsilon)^2}\right\} \qquad (4\text{–}14)$$

where we have defined

$$\epsilon \equiv \frac{E_\gamma}{mc^2} \tag{4–15}$$

In the asymptotic limits,

$$\sigma_c \rightarrow \frac{8}{3}\sigma_0(1-2\epsilon+5.2\epsilon^2-13.3\epsilon^3+\cdots) \qquad \text{for } \epsilon \ll 1 \tag{4–16}$$

and

$$\sigma_c \rightarrow \frac{\sigma_0}{\epsilon}\left[\frac{1}{2}+\ln 2\epsilon\right] \qquad \text{for } \epsilon \gg 1 \tag{4–17}$$

Electrons play the dominant role in the Compton scattering of γ-rays. As we mentioned previously, Compton scattering does not eliminate the γ-ray per se, but will in all probability result in the transfer of some the energy of the γ-ray to the electron, thus absorbing the γ-ray's energy. For γ-rays of energy $E_\gamma \gg mc^2$, almost all of the energy of the γ-ray is absorbed, and then we can consider that the γ-ray has "disappeared."

In some cases it is useful to define an "absorption cross section" σ_a, such that

$$\sigma_a = \left(\frac{\Delta E_\gamma}{E_\gamma}\right)\sigma_c \tag{4–18}$$

where ΔE_γ is the average amount of energy transferred from the γ-ray to the electron. It is then found that (Heitler, 1954)

$$\sigma_a = \epsilon\sigma_c \qquad \text{for } \epsilon \ll 1 \tag{4–19}$$

and

$$\sigma_a = \sigma_c\left[\frac{\ln 2\epsilon - \frac{5}{6}}{\ln 2\epsilon + \frac{1}{2}}\right] \qquad \text{for } \epsilon \gg 1 \tag{4–20}$$

The second type of γ-ray absorption process in matter that we must consider involves the conversion of a γ-ray into an electron-positron pair in the electrostatic field of a charged particle or nucleus. If we designate such a charge field by the symbol CF, such an interaction may be symbolically written as

$$\gamma + \text{CF} \rightarrow e^+ + e^- + \text{CF} \tag{4–21}$$

The conversion interaction, or pair-production as it is usually called, has a cross section that involves an extra factor of the fine structure constant, $\alpha = e^2/hc$, since it involves an intermediate interaction with an electrostatic field. At nonrelativistic energies, this cross section is a complicated function of energy which must be determined numerically.

(See Heitler, 1954, for further details.) However, a closed analytic approximation for this cross section may be given for energies greater than 1 MeV, which corresponds to the energy region where the pair-production cross section becomes more important than the Compton scattering cross section in the determination of the γ-ray mass absorption coefficient for hydrogen gas.

For pair-production in the field of a nucleus of atomic number Z, the cross section for reaction (4–21) is given by

$$\sigma_p = \frac{\alpha}{\pi}\sigma_0\left(\frac{28}{9}\ln 2\epsilon - \frac{218}{27}\right)Z^2 \qquad \text{for } 1 \ll \epsilon \ll \alpha^{-1}Z^{-1/3} \tag{4–22}$$

which is the energy region where electron screening of the nuclear charge field may be neglected. The no-screening case, of course, also holds for an ionized gas (plasma).

In the energy region where the complete screening approximation is valid,

$$\sigma_p = \frac{\alpha}{\pi}\sigma_0\left[\frac{28}{9}\ln\left(183Z^{-1/3}\right) - \frac{2}{27}\right]Z^2 \qquad \text{for } \epsilon \gg \alpha^{-1}Z^{-1/3} \tag{4–23}$$

The threshold energy for pair-production in the field of an atomic nucleus is, of course, $2m_ec^2$. In the case of pair-production in the field

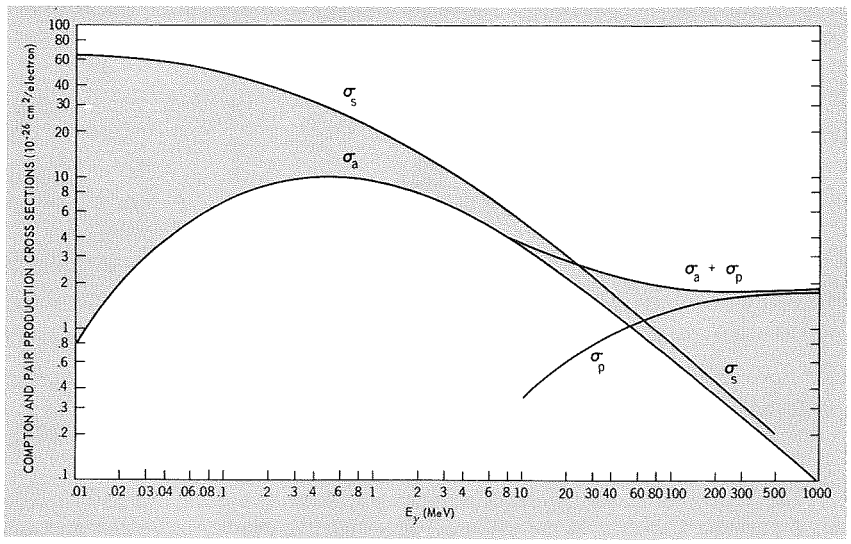

FIGURE 4–2.—Compton scattering (σ_s), Compton absorption (σ_a), pair-production and total ($\sigma_a + \sigma_p$) cross sections as a function of γ-ray energy for absorption of γ-rays in hydrogen gas (based in part on work of Nelmes, 1953).

of atomic electrons, the threshold energy for (4–21) is $4m_e c^2$. Above this energy, the pair-production cross section must be modified to include the additional contribution of the electrons and this may be done approximately by making the replacement

$$Z^2 \rightarrow Z^2(1 + \xi^{-1}Z^{-1}) \tag{4–24}$$

in equations (4–22) and (4–23), where the quantity ξ varies from ≈ 2.6 at $E_\gamma \approx 6.5$ MeV to ≈ 1.2 at $E_\gamma \approx 100$ MeV. For γ-ray energies above 200 MeV, $\xi \approx 1$; and the pair-production cross section has an approximately constant value of 1.8×10^{-26} cm^2, according to the results of Trower (1966). The values of the various cross sections for γ-ray absorption in matter, as discussed in this section, are shown in figure 4–2.

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